

Assignment 4 to 3.2.3.3 - Solution -

Prove that total interest over the useful life according to the residual value method and according to the average value method is identical, if capital invested follows a linear function.

The following symbols are used:

- Cl_0 = Initial investment
- R_n = Residual value at the end of useful life
- n = Useful life
- t = Time
- $Cl(t)$ = Capital invested after t years
- Cl_a = Average capital invested
- i = Interest rate
- I_a = Average interest charge per year
- I = Total interest charge over useful life

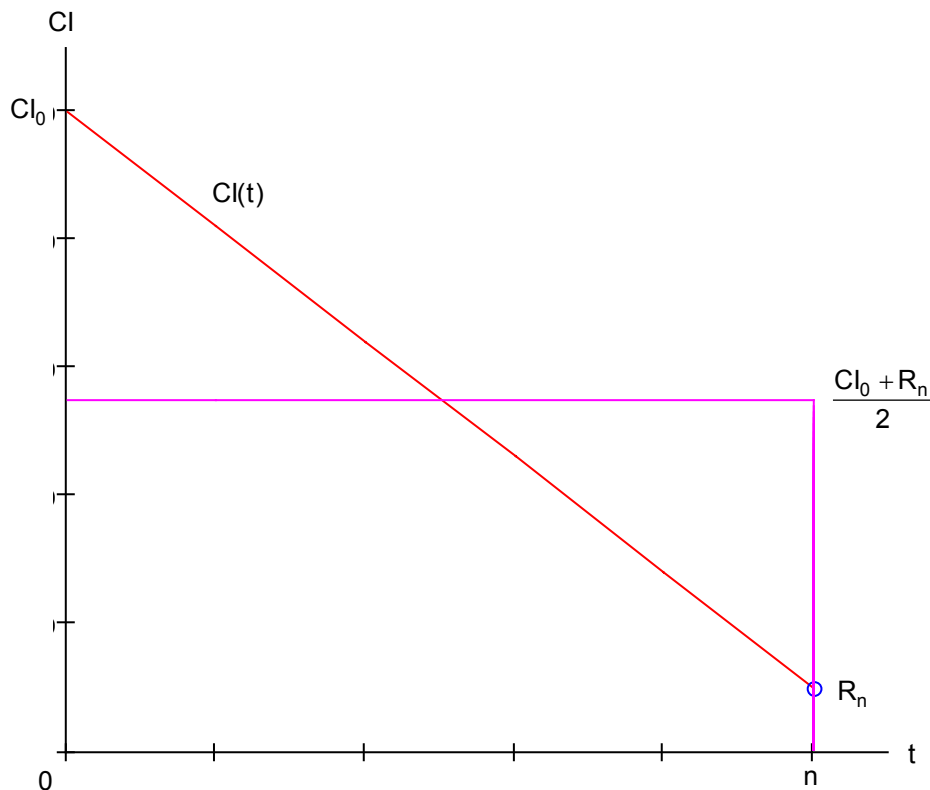
With straight-line depreciation capital invested follows the function:

$$Cl(t) = Cl_0 - \frac{Cl_0 - R_n}{n} \cdot t$$

Average capital invested is:

$$Cl_a = \frac{Cl_0 + R_n}{2}$$

This can be shown graphically as follows:



The average capital invested, multiplied by the interest rate, yields the interest charge per year according to the average value method:

Assignment 4 to 3.2.3.3 - Solution -

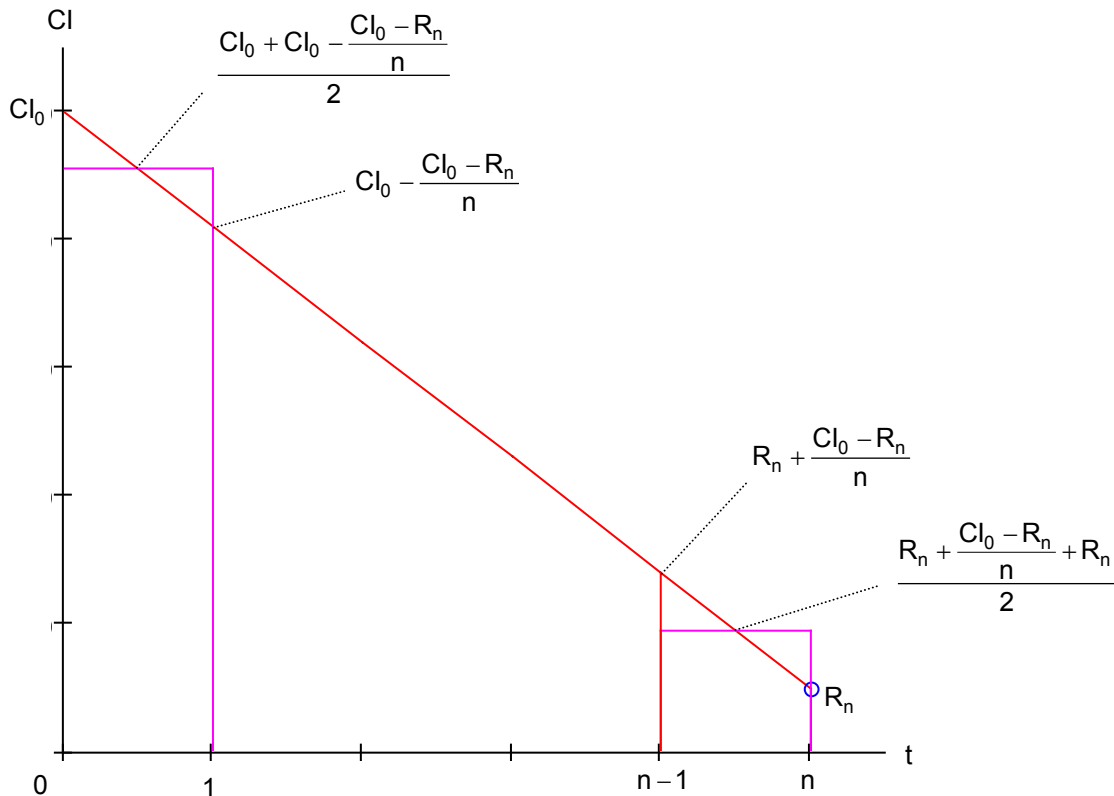
$$I_a = \frac{Cl_0 + R_n}{2} \cdot i$$

Multiplying this amount by the useful life yields the total interest charge:

$$I = \frac{Cl_0 + R_n}{2} \cdot i \cdot n$$

According to the residual value method the interest charge per year is derived from the average capital invested during this year (mean of capital invested at the begin of the year and the value at the end of the year), the so called residual value of that year. This kind of residual value is not to be confused with the residual value R_n at the end of useful life.

The height of the rectangles in the following drawing is the average capital invested in the first year and in the last year respectively:



The average capital invested in subsequent years is decreasing by the depreciation charge of each year. So the amounts of average capital invested constitute an arithmetic row.

The arithmetic mean of this row is the average capital invested over the useful life:

$$Cl_a = \frac{\frac{Cl_0 + Cl_0 - \frac{Cl_0 - R_n}{n}}{2} + \frac{R_n + \frac{Cl_0 - R_n}{n} + R_n}{2}}{2}$$

After some algebraic manipulation:

Assignment 4 to 3.2.3.3
- Solution -

$$CI_a = \frac{CI_0 + R_n}{2}$$

This is the same average capital as determined by the average method .

Accordingly the average interest charge per year an the total interest charge are the same:

$$I_a = \frac{CI_0 + R_n}{2} \cdot i$$

and

$$I = \frac{CI_0 + R_n}{2} \cdot i \cdot n$$

Quod erat demonstrandum.