Cost Function According to the Law of Non-Proportional Returns

$C_f := 98$ Fixed costs

$C_v(x) := x^3 - 12x^2 + 60x$ Variable costs [x = quantity]

$C(x) := C_f + C_v(x)$ Total costs [cost function]

$\frac{d}{dx}C(x) \rightarrow 3 \cdot x^2 - 24 \cdot x + 60$ Marginal costs [first derivative of the cost function]

$C'(x) := \frac{d}{dx}C(x)$

$c_v(x) := \frac{C_v(x)}{x}$ Variable cost per unit

$c_v(x) \rightarrow \frac{x^3 - 12 \cdot x^2 + 60 \cdot x}{x} \text{ simplify } \rightarrow x^2 - 12 \cdot x + 60$

$c(x) := \frac{C(x)}{x}$ Total cost per unit

$c(x) \rightarrow \frac{x^3 - 12 \cdot x^2 + 60 \cdot x + 98}{x} \text{ simplify } \rightarrow \frac{98}{x} - 12 \cdot x + x^2 + 60$

$\lim_{x \to 0^+} c(x) \rightarrow \infty$

$c(x) := \text{if}\left(x > 0, c(x), \lim_{x \to 0^+} c(x)\right)$

$x := 1$ Estimated value of x as a starting point for minimization

$\text{Minimize}(C', x) = 4$ Quantity at which marginal cost is a minimum

$\text{Minimize}(c_v, x) = 6$ Quantity at which variable cost per unit is a minimum

$\text{Minimize}(c, x) = 7$ Quantity at which total cost per unit is a minimum

$x := 0, 0.01 .. 10$ Range of x

$FRAME := 4$ Specific value of x

$\text{tangent}(x) := C(FRAME) - FRAME \cdot C'(FRAME) + C'(FRAME) \cdot x$ Tangent line to the cost function

$\text{secant\_}C_f(x) := C_f + \frac{C(FRAME) - C_f}{FRAME} \cdot x$ Secant line with the cost function, originating from the fixed costs at x = 0

$\text{secant\_0}(x) := \frac{C(FRAME)}{FRAME} \cdot x$ Secant line with the cost function from the origin of the coordinate system

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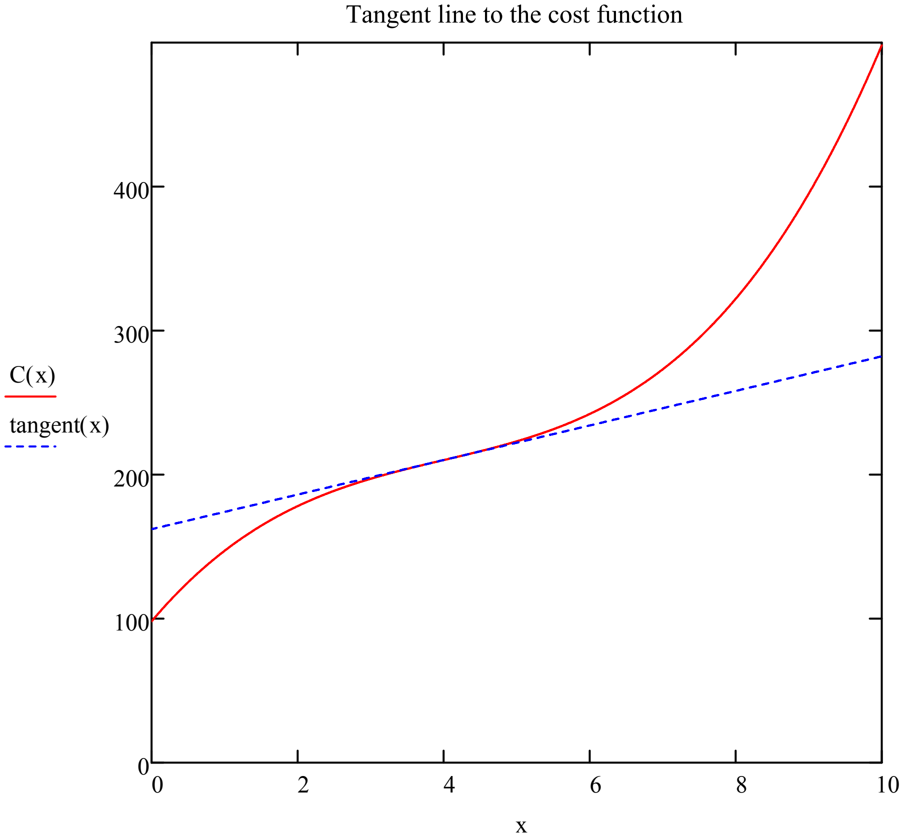

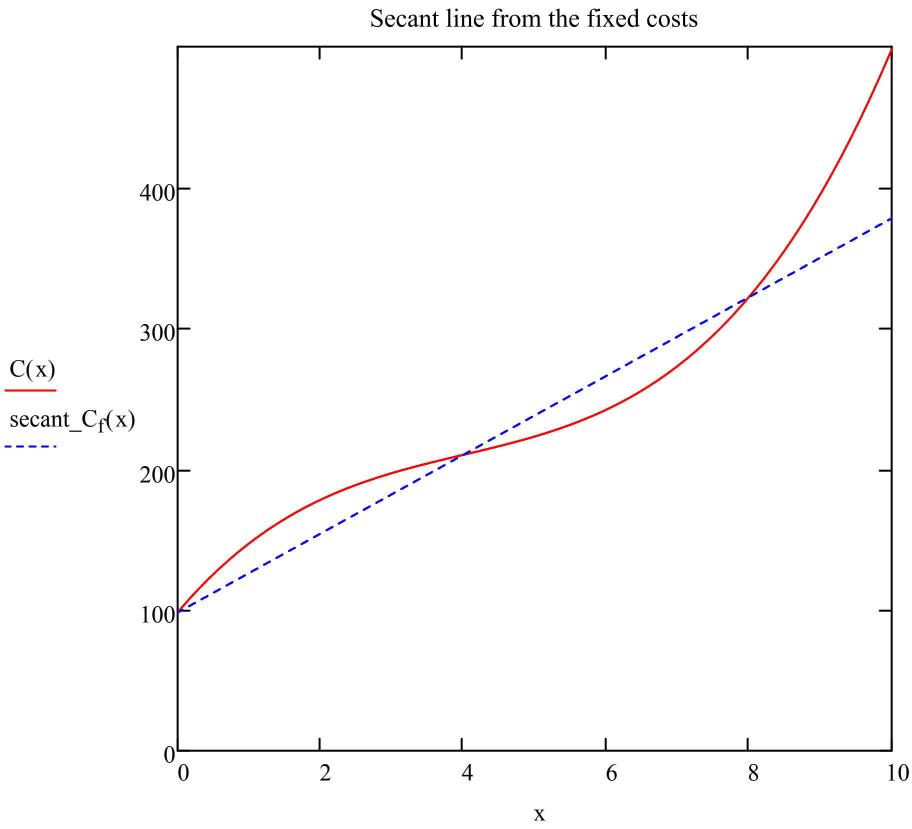

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