Internal Clearing of Services

For internal clearing of services the following data are given:

$$PO = \begin{pmatrix} 1000 \\ 500 \\ 800 \end{pmatrix} \qquad x = \begin{pmatrix} 500 \\ 200 \\ 100 \end{pmatrix} \qquad q = \begin{pmatrix} 70 & 50 & 5 \\ 20 & 40 & 5 \\ 40 & 100 & 20 \end{pmatrix} \qquad c = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad ORIGIN = 1$$

The vector PO represents the primary overheads of each indirect cost centre (ICC):

$PO_1 = 1000$ Primary overheads of ICC 1

$PO_2 = 500$ Primary overheads of ICC 2

$PO_3 = 800$ Primary overheads of ICC 3

The vector x contains the output of the ICCs, measured by units of quantity (QU):

$x_1 = 500$ Output of ICC 1 [QU]

$x_2 = 200$ Output of ICC 2 [QU]

$x_3 = 100$ Output of ICC 3 [QU]

The lines of matrix q show the deliveries of these outputs to each ICC, and the columns show the supplying ICC:

$q_{1,1} = 70$ Deliveries to ICC 1 by ICC 1 [QU]

$q_{1,2} = 50$ Deliveries to ICC 1 by ICC 2 [QU]

$q_{1,3} = 5$ Deliveries to ICC 1 by ICC 3 [QU]

$q_{2,1} = 20$ Deliveries to ICC 2 by ICC 1 [QU]

$q_{2,2} = 40$ Deliveries to ICC 2 by ICC 2 [QU]

$q_{2,3} = 5$ Deliveries to ICC 2 by ICC 3 [QU]

$q_{3,1} = 40$ Deliveries to ICC 3 by ICC 1 [QU]

$q_{3,2} = 100$ Deliveries to ICC 3 by ICC 2 [QU]

$q_{3,3} = 20$ Deliveries to ICC 3 by ICC 3 [QU]

The problem is to determine the cost per unit for the output of each indirect cost centre, c_1, c_2, c_3. At the start, the vector c contains estimated values for these costs per unit. These values are arbitrary, they are only necessary as an initial value for the solution algorithm.

$c_1 = 1$ Estimated cost per unit for the output of ICC 1

$c_2 = 1$ Estimated cost per unit for the output of ICC 2

$c_3 = 1$ Estimated cost per unit for the output of ICC 3

The variable ORIGIN determines the first digit of the fields.

In order to determine c_1, c_2 and c_3, it is necessary to establish sets of equations following the guidelines of the specific method for internal clearing of services applied.

However, the basic definitions of the left sides and the right sides in all sets of equations are identical: The left sides of the following equations represent the total cost of each indirect cost centre after internal clearing of services between indirect cost centres took place, i.e. the costs of the ICCs were increased for their consumption of services of ICCs and the costs were decreased for their own deliveries to ICCs. The result of this internal clearing of services between indirect cost centres is for each indirect cost centre the cost to be charged to direct cost centres.

Whereas the left sides of the equations represent the costs to be charged to direct cost centres, the right sides represent the costs actually charged to them. Of course, both - the cost to be charged to direct cost centres and the costs actually charged to them - must be identical for every ICC.

The basis for charging direct cost centres is the total output delivered to them. This amount can be determined by deducting from the total output of an ICC all items consumed by ICCs, be it the ICC itself or other ICCs. The rest was delivered to direct cost centres. If this amount is multiplied by the appropriate cost per unit, the result must be the cost to be charged to the direct cost centres, i.e. the left side of the equation.

No internal clearing of services between indirect cost centres

As the name says, according to this method no internal clearing of services between indirect cost centres takes place. So every ICC has to charge its primary overheads to the direct cost centres. As mentioned before, the basis for this charge is the number of units actually delivered to direct cost centres. So we have:

Given

$$PO_1 = \left(x_1 - q_{1,1} - q_{2,1} - q_{3,1}\right) \cdot c_1$$

$$PO_2 = \left(x_2 - q_{1,2} - q_{2,2} - q_{3,2}\right) \cdot c_2$$

$$PO_3 = \left(x_3 - q_{1,3} - q_{2,3} - q_{3,3}\right) \cdot c_3$$

$$c = \mathrm{Find}(c)$$

$$c = \begin{pmatrix} 2.70 \\ 50.00 \\ 11.43 \end{pmatrix}$$

In order to resolve above equations manually, they have to be rearranged for the costs per unit:

$$c_1 = \frac{PO_1}{x_1 - q_{1,1} - q_{2,1} - q_{3,1}}$$

$$c_1 = 2.70$$

$$c_2 = \frac{PO_2}{x_2 - q_{1,2} - q_{2,2} - q_{3,2}}$$

$$c_2 = 50.00$$

$$c_3 = \frac{PO_3}{x_3 - q_{1,3} - q_{2,3} - q_{3,3}}$$

$$c_3 = 11.43$$

In order to check whether all primary costs of ICCs were transferred to direct cost centres, the vector CC (cost charge) was established:

$$CC = \begin{bmatrix} c_1 \cdot \left(x_1 - q_{1,1} - q_{2,1} - q_{3,1}\right) \\ c_2 \cdot \left(x_2 - q_{1,2} - q_{2,2} - q_{3,2}\right) \\ c_3 \cdot \left(x_3 - q_{1,3} - q_{2,3} - q_{3,3}\right) \end{bmatrix} \qquad CC = \begin{pmatrix} 1000 \\ 500 \\ 800 \end{pmatrix} \qquad \sum CC = 2300$$

With the concrete numbers of the example following equations for the costs per unit have to be resolved:

$$c_1 = \frac{1000}{500 - 70 - 20 - 40}$$

$$c_2 = \frac{500}{200 - 50 - 40 - 100}$$

$$c_3 = \frac{800}{100 - 5 - 5 - 20}$$

Unidirectional internal clearing of services

According to unidirectional internal clearing of services every ICC charges all subsequent ICCs for its deliveries to them, but the ICC is not charged for its consumption of services rendered by subsequent ICCs. Furthermore, it is not charged for its consumption of its own output. Every ICC is only charged by previous cost centres.

In other words: ICC 1 charges ICC 2 and ICC 3 for their consumption of its output. ICC 1 is not charged for its consumption of its own output neither for that of others (although all this takes place). ICC 2 is charged by ICC 1 and it charges the subsequent ICC 3. ICC 3 is only charged by previous cost centres, but it does not charge them.

Thus the following equations are to be resolved:

Given

$$PO_1 - q_{2,1} \cdot c_1 - q_{3,1} \cdot c_1 = \left(x_1 - q_{1,1} - q_{2,1} - q_{3,1}\right) \cdot c_1$$

$$PO_2 + q_{2,1} \cdot c_1 - q_{3,2} \cdot c_2 = \left(x_2 - q_{1,2} - q_{2,2} - q_{3,2}\right) \cdot c_2$$

$$PO_3 + q_{3,1} \cdot c_1 + q_{3,2} \cdot c_2 = \left(x_3 - q_{1,3} - q_{2,3} - q_{3,3}\right) \cdot c_3$$

$$c = \text{Find}(c)$$

$$c = \begin{pmatrix} 2.33 \\ 4.97 \\ 19.86 \end{pmatrix}$$

Rearranging these equations for the costs per unit:

$$c_1 = \frac{PO_1}{x_1 - q_{1,1}}$$

$$c_1 = 2.33$$

$$c_2 = \frac{PO_2 + c_1 \cdot q_{2,1}}{x_2 - q_{1,2} - q_{2,2}}$$

$$c_2 = 4.97$$

$$c_3 = \frac{PO_3 + q_{3,1} \cdot c_1 + q_{3,2} \cdot c_2}{x_3 - q_{1,3} - q_{2,3} - q_{3,3}}$$

$$c_3 = 19.86$$

Checking total costs charged to direct cost centres:

$$CC = \begin{bmatrix} c_1 \cdot \left(x_1 - q_{1,1} - q_{2,1} - q_{3,1}\right) \\ c_2 \cdot \left(x_2 - q_{1,2} - q_{2,2} - q_{3,2}\right) \\ c_3 \cdot \left(x_3 - q_{1,3} - q_{2,3} - q_{3,3}\right) \end{bmatrix} \qquad CC = \begin{pmatrix} 860.47 \\ 49.68 \\ 1389.85 \end{pmatrix} \qquad \sum CC = 2300$$

With the concrete numbers of the example the costs per unit are:

$$c_1 = \frac{1000}{500 \quad - \quad 70}$$

$$c_2 = \frac{500 \quad + \quad 2{,}3255814 \quad \cdot \quad 20}{200 \quad - \quad 50 \quad - \quad 40}$$

$$c_3 = \frac{800 \quad + \quad 40 \quad \cdot \quad 2{,}3255814 \quad + \quad 100 \quad \cdot \quad 4{,}96828753}{100 \quad - \quad 5 \quad - \quad 5 \quad - \quad 20}$$

Mutual internal clearing of services

According to mutual internal clearing of services all ICCs are charged for their consumption, whosoever was the supplier, and all ICCs are discharged from the costs of all their deliveries:

ICC 1 is charged for its consumption of its own output and, of course, for its consumption of output produced by ICC 2 and ICC 3. On the other hand, ICC 1 is discharged for its own supplies to itself and for its deliveries to ICC 2 and ICC 3.

ICC 2 is charged by ICC 1, by itself and by ICC 3. It is discharged for its deliveries to ICC 1, to itself and to ICC 3.

ICC 3 is charged by ICC 1, by ICC 2 and by itself. It is discharged for its deliveries to ICC 1, to ICC 2 and to itself.

Thus the following set of equations must be resolved:

Given

$$PO_1 + q_{1,1} \cdot c_1 + q_{1,2} \cdot c_2 + q_{1,3} \cdot c_3 - q_{1,1} \cdot c_1 - q_{2,1} \cdot c_1 - q_{3,1} \cdot c_1 = \left(x_1 - q_{1,1} - q_{2,1} - q_{3,1}\right) \cdot c_1$$

$$PO_2 + q_{2,1} \cdot c_1 + q_{2,2} \cdot c_2 + q_{2,3} \cdot c_3 - q_{1,2} \cdot c_2 - q_{2,2} \cdot c_2 - q_{3,2} \cdot c_2 = \left(x_2 - q_{1,2} - q_{2,2} - q_{3,2}\right) \cdot c_2$$

$$PO_3 + q_{3,1} \cdot c_1 + q_{3,2} \cdot c_2 + q_{3,3} \cdot c_3 - q_{1,3} \cdot c_3 - q_{2,3} \cdot c_3 - q_{3,3} \cdot c_3 = \left(x_3 - q_{1,3} - q_{2,3} - q_{3,3}\right) \cdot c_3$$

$$c = \text{Find}(c)$$

$$c = \begin{pmatrix} 2.98 \\ 4.01 \\ 16.51 \end{pmatrix}$$

Above equations may be simplified:

Given

$$PO_1 + c_1 \cdot q_{1,1} + c_2 \cdot q_{1,2} + c_3 \cdot q_{1,3} = c_1 \cdot x_1$$

$$PO_2 + c_1 \cdot q_{2,1} + c_2 \cdot q_{2,2} + c_3 \cdot q_{2,3} = c_2 \cdot x_2$$

$$PO_3 + c_1 \cdot q_{3,1} + c_2 \cdot q_{3,2} + c_3 \cdot q_{3,3} = c_3 \cdot x_3$$

$$c = \text{Find}(c)$$

$$c = \begin{pmatrix} 2.98 \\ 4.01 \\ 16.51 \end{pmatrix}$$

These simplified equations show a normal understanding of costs: The left side is the primary cost plus all secondary costs, i.e. the total cost necessary to produce the total output. For each ICC the secondary costs consist in its consumption of output produced by ICC 1, priced at c_1, of output produced by ICC 2, priced at c_2, of output produced by ICC 3, priced at c_3. These total costs of each ICC were necessary to produce the ICC's output; and these costs have to be covered by the appropriate cost rate, i.e. the cost per unit, multiplied by total output, is equal to these costs - the right side of the equations.

Using the concrete figures, the following set of equations is to be solved:

$$\begin{aligned}
1000 + 70\ c_1 + 50\ c_2 + 5\ c_3 &= 500\ c_1 \\
500 + 20\ c_1 + 40\ c_2 + 5\ c_3 &= 200\ c_2 \\
800 + 40\ c_1 + 100\ c_2 + 20\ c_3 &= 100\ c_3
\end{aligned}$$

If matrix notation is used, the problem can be further simplified:

Given

$$PO + q \cdot c = \begin{pmatrix} x_1 \cdot c_1 \\ x_2 \cdot c_2 \\ x_3 \cdot c_3 \end{pmatrix}$$

$$c = \text{Find}(c)$$

$$c = \begin{pmatrix} 2.98 \\ 4.01 \\ 16.51 \end{pmatrix}$$

However, rearranging the equations for the cost per unit gives more economic insights:

Given

$$c_1 = \frac{PO_1 + c_1 \cdot q_{1,1} + c_2 \cdot q_{1,2} + c_3 \cdot q_{1,3}}{x_1}$$

$$c_2 = \frac{PO_2 + c_1 \cdot q_{2,1} + c_2 \cdot q_{2,2} + c_3 \cdot q_{2,3}}{x_2}$$

$$c_3 = \frac{PO_3 + c_1 \cdot q_{3,1} + c_2 \cdot q_{3,2} + c_3 \cdot q_{3,3}}{x_3}$$

$$c = \text{Find}(c)$$

$$c = \begin{pmatrix} 2.98 \\ 4.01 \\ 16.51 \end{pmatrix}$$

Again, a normal understanding of costs is visible here, since cost per unit is simply total cost (primary and secondary cost) divided by total output. This is possible by a comprehensive modelling of all relationships between ICCs. Contrary to that, the other methods deduct in the denominator all quantities not charged to ICCs. By doing this, those quantitities, not taken into account in the internal clearing of service between ICCs, will increase the cost rate and so they are taken into account in this indirect way.

Clearly it is preferable to model all relationships as exactly as possible, as does the mutual internal clearing of services. These cost rates are the most exact ones. Nevertheless, at the end all primary costs of the ICCs are transferred to the direct cost centres and turn to be secondary costs there:

$$CC = \begin{bmatrix} c_1 \cdot \left(x_1 - q_{1,1} - q_{2,1} - q_{3,1}\right) \\ c_2 \cdot \left(x_2 - q_{1,2} - q_{2,2} - q_{3,2}\right) \\ c_3 \cdot \left(x_3 - q_{1,3} - q_{2,3} - q_{3,3}\right) \end{bmatrix} \qquad CC = \begin{pmatrix} 1104.19 \\ 40.14 \\ 1155.67 \end{pmatrix} \qquad \sum CC = 2300$$

Legend:

PO_i	=	Primary overheads in indirect cost centre (ICC) i
i	=	1..n [Index of indirect cost centres in rows]
n	=	Number of indirect cost centres
x_i	=	Output of ICC i
$q_{i,j}$	=	Quantity of output charged to ICC i by ICC j
j	=	1..n [Index of indirect cost centres in columns]
c_i	=	Cost per unit in ICC i
CC_i	=	Costs charged to direct cost centres by ICC i