Internal Clearing of Services

For internal clearing of services the following data are given:

$$PO := \begin{pmatrix} 1000 \\ 500 \\ 800 \end{pmatrix} \qquad x := \begin{pmatrix} 500 \\ 200 \\ 100 \end{pmatrix} \qquad q := \begin{pmatrix} 70 & 50 & 5 \\ 20 & 40 & 5 \\ 40 & 100 & 20 \end{pmatrix} \qquad c := \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad ORIGIN \equiv 1$$

The vector PO represents the primary overheads of each indirect cost centre (ICC):

$PO_1 = 1000$ Primary overheads of ICC 1

$PO_2 = 500$ Primary overheads of ICC 2

$PO_3 = 800$ Primary overheads of ICC 3

The vector x contains the output of the ICCs, measured by units of quantity (QU):

$x_1 = 500$ Output of ICC 1 [QU]

$x_2 = 200$ Output of ICC 2 [QU]

$x_3 = 100$ Output of ICC 3 [QU]

The lines of matrix q show the deliveries of these outputs to each ICC, and the columns show the supplying ICC:

$q_{1,1} = 70$ Deliveries to ICC 1 by ICC 1 [QU]

$q_{1,2} = 50$ Deliveries to ICC 1 by ICC 2 [QU]

$q_{1,3} = 5$ Deliveries to ICC 1 by ICC 3 [QU]

$q_{2,1} = 20$ Deliveries to ICC 2 by ICC 1 [QU]

$q_{2,2} = 40$ Deliveries to ICC 2 by ICC 2 [QU]

$q_{2,3} = 5$ Deliveries to ICC 2 by ICC 3 [QU]

$q_{3,1} = 40$ Deliveries to ICC 3 by ICC 1 [QU]

$q_{3,2} = 100$ Deliveries to ICC 3 by ICC 2 [QU]

$q_{3,3} = 20$ Deliveries to ICC 3 by ICC 3 [QU]

The problem is to determine the cost per unit for the output of each indirect cost centre, c_1, c_2, c_3. At the start, the vector c contains estimated values for these costs per unit. These values are arbitrary, they are only necessary as an initial value for the solution algorithm.

$c_1 = 1$ Estimated cost per unit for the output of ICC 1

$c_2 = 1$ Estimated cost per unit for the output of ICC 2

$c_3 = 1$ Estimated cost per unit for the output of ICC 3

The variable ORIGIN determines the first digit of the fields.

Internal Clearing of Services

In order to have a general view on the problem, the following definitions are made:

$n := 3$	Number of ICCs
$i := 1..n$	Subscript for ICCs
$j := 1..n$	Subscript for ICCs
$q_{i,j}$	Deliveries to ICC i by ICC j
$q_{j,i}$	Deliveries to ICC j by ICC i

No internal clearing of services between indirect cost centres

The cost per unit for the output of indirect cost centre i is:

$$c_i := \frac{PO_i}{x_i - \sum_j q_{j,i}}$$

$c_i =$

2.70
50.00
11.43

Formulated explicitly for three cost centres:

Given

$$c_1 = \frac{PO_1}{x_1 - q_{1,1} - q_{2,1} - q_{3,1}}$$

$$c_2 = \frac{PO_2}{x_2 - q_{1,2} - q_{2,2} - q_{3,2}}$$

$$c_3 = \frac{PO_3}{x_3 - q_{1,3} - q_{2,3} - q_{3,3}}$$

$c := \text{Find}(c)$

$$c = \begin{pmatrix} 2.70 \\ 50.00 \\ 11.43 \end{pmatrix}$$

Unidirectional internal clearing of services

The cost per unit for the output of indirect cost centre i is:

$$c_i := \mathrm{if}\left[(i) = 1, \frac{PO_1}{x_1 - q_{1,1}}, \frac{PO_i + \sum_{j=1}^{i-1} c_j \cdot q_{i,j}}{x_i - \sum_{j=1}^{i} q_{j,i}}\right]$$

$c_i =$

2.33
4.97
19.86

Formulated explicitly for three cost centres:

Given

$$c_1 = \frac{PO_1}{x_1 - q_{1,1}}$$

$$c_2 = \frac{PO_2 + c_1 \cdot q_{2,1}}{x_2 - q_{1,2} - q_{2,2}}$$

$$c_3 = \frac{PO_3 + c_1 \cdot q_{3,1} + c_2 \cdot q_{3,2}}{x_3 - q_{1,3} - q_{2,3} - q_{3,3}}$$

$c := \mathrm{Find}(c)$

$$c = \begin{pmatrix} 2.33 \\ 4.97 \\ 19.86 \end{pmatrix}$$

Mutual internal clearing of services

The cost per unit for the output of indirect cost centre i is:

$$c_i = \frac{PO_i + \sum_j c_j \cdot q_{i,j}}{x_i}$$

Since the costs per unit of *all* indirect cost centres appear on the right side of this equation, the costs per unit must be determined simultaneously. For doing this it is necessary to formulate the set of equations explicitly:

Given

$$c_1 = \frac{PO_1 + c_1 \cdot q_{1,1} + c_2 \cdot q_{1,2} + c_3 \cdot q_{1,3}}{x_1}$$

$$c_2 = \frac{PO_2 + c_1 \cdot q_{2,1} + c_2 \cdot q_{2,2} + c_3 \cdot q_{2,3}}{x_2}$$

$$c_3 = \frac{PO_3 + c_1 \cdot q_{3,1} + c_2 \cdot q_{3,2} + c_3 \cdot q_{3,3}}{x_3}$$

$c := \text{Find}(c)$

$$c = \begin{pmatrix} 2.98 \\ 4.01 \\ 16.51 \end{pmatrix}$$

In matrix notation:

Given

$$PO + q \cdot c = \begin{pmatrix} x_1 \cdot c_1 \\ x_2 \cdot c_2 \\ x_3 \cdot c_3 \end{pmatrix}$$

$c := \text{Find}(c)$

$$c = \begin{pmatrix} 2.98 \\ 4.01 \\ 16.51 \end{pmatrix}$$